Proposed ICCF Rating Algorithm

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The proposed ICCF system is designed as an extension of the Glicko rating system. [1] The Glicko system was developed to address a fundamental problem with the Elo system, specifically that the relative reliability of ratings were not incorporated into rating calculations in any formalized manner. For example, if an opponent's rating is unreliable because they have only played a small number of rated games, one's own rating should not change appreciably based on the result of a game. Analogously, results against opponents with reliable ratings should correspond to potentially larger rating changes. The Glicko system formalized this concept by the computation of an RD (ratings deviation), more commonly known in Statistics as a "standard deviation." The larger the RD, the more uncertain or unreliable the player's rating. Every player in the Glicko system has a rating and an RD that get updated in the rating calculations.

The extension developed for ICCF explicitly models the individual probabilities of a win, draw or loss, and recognizes that the probability of a draw between strong players is higher than the probability of a draw between weaker players. The derivation of this extended system is similar to the derivation of the Glicko rating system [2] By acknowledging the difference in the probability of drawn games at different rating levels, the rating changes for higher rated players are not impacted as much by the rating of their opponents compared to weaker players.

Rating algorithm:

The following steps are to be computed in parallel for all players. The procedure assumes that every player at the start of the current rating period either has a rating and RD at the end of a previous rating period, or that a player is unrated. The following steps determine

[1] The details of the Glicko rating system derivation can be found in Glickman, M. E. (1999). Parameter estimation in large dynamic paired comparison experiments. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 48(3), 377-394.

[2] Specifically, given the form of the probabilities of a win, draw and loss, and a normal prior distribution for the player centered at their rating and with the RD as the standard deviation, the log-posterior density for a player's rating (integrating over the prior distributions of the opponents' ratings via a 2-point Gauss-Hermite quadrature) is approximately maximized using a 1-step Newton-Raphson iteration. This requires the calculation of the gradient (first derivative) and Hessian (second derivative) at the player's pre-period rating.

the rating and RD at the end of the current rating period, and these are used as the starting point for the next rating period.

1. Determine the rating and RD for each player at the start of the new rating period based on their rating and RD at the end of the previous period. For each player:

   (a) If the player is unrated, set the initial rating to 1800 (assuming no other external rating such as a FIDE rating), and set the RD to 250.

   (b) If the player is rated, use the player's rating from the end of the last period, and calculate the new RD from the value at the last period (RD_{old}).

   - If $RD_{old} > 120$, then $RD = RD_{old}$.
   - If $RD_{old} \leq 120$, then $RD = \sqrt{RD_{old}^2 + c^2}$ where $c = 25$ is a constant that accounts for the increase in rating uncertainty between rating periods.

2. Carry out the following updating calculations for each player separately:

   For a specific player, assume that their pre-period rating is r, and the ratings deviation is RD determined from Step 1. Suppose the player competes against m opponents during the rating period. Let the pre-period ratings of the opponents (again from Step 1) be $r_1, r_2, \ldots, r_m$ and the ratings deviations be $RD_1, RD_2, \ldots, RD_m$. Also let $y_1, \ldots, y_m$ be the outcome against each opponent, with an outcome being either 1, 0.5, or 0 for a win, draw and loss.[3] Note that multiple games against the same opponent are treated as games against multiple opponents with the same rating and RD. Let r' and RD' denote the post-period rating and ratings deviation for the player. The rating algorithm involves the following steps.

   (a) Convert the ratings and RDs to the standardized scale:

   $$\mu = (r - 1500)/173.7, \qquad \sigma = RD/173.7$$
   $$\mu_j = (r_j - 1500)/173.7, \qquad \sigma_j = RD_j/173.7$$

   for $j = 1, \ldots, m$. The value 173.7 is $400/\log_e(10)$, and is used to convert between the Elo winning expectancy $1/(1 + 10^{(r-r_j)/400})$ against opponent j to the standardized logistic probability $1/(1 + e^{(\mu - \mu_j)})$.

   (b) Set the system parameter values: $\alpha_0 = 0, \alpha_1 = 0, \beta_0 = 1.0986, \beta_1 = 0.17037$. [4] These values were determined so that two players rated 1500 would have a 60% probability of a draw, and two players rated 2500 would have an 80% probability of a draw. [5]

[3] For the variation of the system that accounts for color, these outcome values change slightly.

[4] Note that α_0 and α_1 are values relevant to a version of this system that accounts for white advantage, but set to 0 when color is not accounted for in the rating system.

[5] In fact, these probabilities are somewhat conservative because the empirical rates of draws among top players is even higher than 80%. The slightly lower assumed probability accounts for the possibility that some high-rated players do not draw as frequently, and that by assuming a lower draw probability rating changes can depend more on the rating of the opponent.

(c) Define the probability of a win, draw and loss as a function of the standardized ratings μ and μ_j as

$$\begin{aligned}\Pr(\text{win}|\mu,\mu_j) &= \exp(\mu + x_j(\alpha_0+\alpha_1\bar{\mu})/4)/S\\ \Pr(\text{draw}|\mu,\mu_j) &= \exp(\beta_0 + (1+\beta_1)\bar{\mu})/S\\ \Pr(\text{loss}|\mu,\mu_j) &= \exp(\mu_j - x_j(\alpha_0+\alpha_1\bar{\mu})/4)/S\end{aligned}$$

where $\bar{\mu} = (\mu+\mu_j)/2$, $x_j = 1$ if the player has white and $x_j = -1$ if the player has black (not relevant unless α_0 or α_1 are different from 0), and

$$S = \exp(\mu + x_j(\alpha_0+\alpha_1\bar{\mu})/4) + \exp(\beta_0+(1+\beta_1)\bar{\mu}) + \exp(\mu_j - x_j(\alpha_0+\alpha_1\bar{\mu})/4).$$

For the system without a white advantage, these simplify to

$$\begin{aligned}\Pr(\text{win}|\mu,\mu_j) &= \exp(\mu)/S\\ \Pr(\text{draw}|\mu,\mu_j) &= \exp(\beta_0 + (1+\beta_1)\bar{\mu})/S\\ \Pr(\text{loss}|\mu,\mu_j) &= \exp(\mu_j)/S,\end{aligned}$$

and

$$S = \exp(\mu) + \exp(\beta_0+(1+\beta_1)\bar{\mu}) + \exp(\mu_j).$$

(d) To account for the uncertainty in an opponent's rating in computing probabilities, the probabilities in step (c) are replaced with the average of probabilities evaluated at the opponent's rating of $\mu_j - \sigma_j$ and $\mu_j + \sigma_j$. Define

$$\begin{aligned}Pw_j^- &= \Pr(\text{win}|\mu,\mu_j-\sigma_j),\\ Pw_j^+ &= \Pr(\text{win}|\mu,\mu_j+\sigma_j),\\ Pd_j^- &= \Pr(\text{draw}|\mu,\mu_j-\sigma_j),\\ Pd_j^+ &= \Pr(\text{draw}|\mu,\mu_j+\sigma_j),\\ Pl_j^- &= \Pr(\text{loss}|\mu,\mu_j-\sigma_j),\\ Pl_j^+ &= \Pr(\text{loss}|\mu,\mu_j+\sigma_j)\end{aligned}$$

which involves replacing μ_j in the formulae in step (c) with either $\mu_j-\sigma_j$ or $\mu_j+\sigma_j$ as specified above. Now let

$$P_j = \begin{cases} Pw_j^- + Pw_j^+ & \text{if } y_j = 1\\ Pd_j^- + Pd_j^+ & \text{if } y_j = 0.5\\ Pl_j^- + Pl_j^+ & \text{if } y_j = 0\end{cases}$$

which is twice the average probabilities of the observed game result against opponent j. This approach to accounting for the uncertainty by averaging two probabilities is based on a 2-point Gauss-Hermite quadrature.

(e) Let

$$\begin{aligned}w_{1j}^- &= Pw_j^- + 0.5Pd_j^-,\\ w_{1j}^+ &= Pw_j^+ + 0.5Pd_j^+,\\ w_{2j}^- &= Pw_j^- + 0.25Pd_j^-,\\ w_{2j}^+ &= Pw_j^+ + 0.25Pd_j^+\end{aligned}$$

Now let[6]

$$D_{1j} = \begin{cases} (Pw_j^-(1 - w_{1j}^-) + Pw_j^+(1 - w_{1j}^+))/P_j & \text{if } y_j = 1 \\ (Pd_j^-(0.5 - w_{1j}^-) + Pd_j^+(0.5 - w_{1j}^+))/P_j & \text{if } y_j = 0.5 \\ (Pl_j^-(0 - w_{1j}^-) + Pl_j^+(0 - w_{1j}^+))/P_j & \text{if } y_j = 0 \end{cases}$$

Also let[7]

$$D_{2j} = \begin{cases} \big(Pw_j^-(1 - w_{2j}^- + 2w_{1j}^-(w_{1j}^- - 1)) + \\ Pw_j^+(1 - w_{2j}^+ + 2w_{1j}^+(w_{1j}^+ - 1))\big)/P_j - D_{1j}^2 & \text{if } y_j = 1 \\ \big(Pd_j^-(0.25 - w_{2j}^- + 2w_{1j}^-(w_{1j}^- - 0.5)) + \\ Pd_j^+(0.25 - w_{2j}^+ + 2w_{1j}^+(w_{1j}^+ - 0.5))\big)/P_j - D_{1j}^2 & \text{if } y_j = 0.5 \\ \big(Pl_j^-(0 - w_{2j}^- + 2w_{1j}^-(w_{1j}^- - 0)) + \\ Pl_j^+(0 - w_{2j}^+ + 2w_{1j}^+(w_{1j}^+ - 0))\big)/P_j - D_{1j}^2 & \text{if } y_j = 0 \end{cases}$$

(f) Now compute the updated ratings on the standardized scale: [8]

$$\sigma' = \sqrt{\frac{1}{1/\sigma^2 - \sum_{j=1}^{m} D_{2j}}}$$

$$\mu' = \mu + (\sigma')^2 \sum_{j=1}^{m} D_{1j}$$

(g) Finally, convert μ' and σ' to the Elo scale:

$$r' = 173.7\mu' + 1500,$$

$$RD' = 173.7\sigma'$$

These are the post-period rating and RD for the player based on game results. If the player has not had any game results in the current period, the rating and RD remain unchanged from the starting rating and RD.

Example calculation:

To demonstrate the calculations for updating a rating in actual practice, suppose a player rated 1900 with an RD of 80 finishes games against only three opponents during a rating

[6] This computation is for the gradient of the log-posterior density.

[7] This computation is for the Hessian of the log-posterior density.

[8] The computation for μ' is the one-step Newton-Raphson update, using the information determined to compute σ'.

period with the following results.

Opponent	Rating	RD	Result against Opponent
1	1750	150	Win
2	2000	70	Draw
3	2300	50	Loss

The computations below are carried out to machine-accuracy, but some intermediate calculations are displayed to lesser precision for easy viewing. From the above information, we have $r = 1900$, $RD = 80$, and

- $r_1 = 1750$, $r_2 = 2000$, and $r_3 = 2300$
- $RD_1 = 150$, $RD_2 = 70$, and $RD_3 = 50$.

The player's rating is updated as follows:

(a) The ratings and RDs are converted to the standardized scale (rounded to the ten-thousandth digit):

$$\mu = (r - 1500)/173.7 = (1900 - 1500)/173.7 = 2.3028, \qquad \sigma = RD/173.7 = 80/173.7 = 0.4606$$
$$\mu_1 = (r_1 - 1500)/173.7 = (1750 - 1500)/173.7 = 1.4393, \qquad \sigma_1 = RD_1/173.7 = 150/173.7 = 0.8636$$
$$\mu_2 = (r_2 - 1500)/173.7 = (2000 - 1500)/173.7 = 2.8785, \qquad \sigma_2 = RD_2/173.7 = 70/173.7 = 0.4030$$
$$\mu_3 = (r_3 - 1500)/173.7 = (2300 - 1500)/173.7 = 4.6056, \qquad \sigma_2 = RD_3/173.7 = 50/173.7 = 0.2879$$

(b) Use the system parameters as set in the algorithm description.

(c) Use the formula definitions for computing probabilities of wins, draws and losses as set in the algorithm description above.

(d) Using the formulas in step (c) in the algorithm, determine the following values (rounded to the thousandth digit):

Opponent	Pw_j^-	Pw_j^+	Pd_j^-	Pd_j^+	Pl_j^-	Pl_j^+	y_j	P_j
1	0.358	0.155	0.578	0.690	0.064	0.155	1.0	$0.358 + 0.155 = 0.513$
2	0.141	0.087	0.692	0.683	0.167	0.231	0.5	$0.692 + 0.683 = 1.374$
3	0.044	0.029	0.629	0.585	0.327	0.386	0.0	$0.327 + 0.386 = 0.713$

(e) From the above table, compute using the formulas in the algorithm (rounded to the ten thousandth digit):

Opponent	w_{1j}^-	w_{1j}^+	w_{2j}^-	w_{2j}^+
1	0.6471	0.5000	0.5025	0.3276
2	0.4867	0.4280	0.3138	0.2573
3	0.3583	0.3215	0.2010	0.1752

D_{1j} and D_{2j} are computed as shown below for each j (rounding to the nearest hundred-thousandth digit):

For $j = 1$, $y_j = 1$, so that

$$\begin{aligned} D_{1j} &= (Pw_j^-(1 - w_{1j}^-) + Pw_j^+(1 - w_{1j}^+))/P_j \\ &= 0.39739 \end{aligned}$$

and

$$\begin{aligned} D_{2j} &= \left(Pw_j^-(1 - w_{2j}^- + 2w_{1j}^-(w_{1j}^- - 1)) + Pw_j^+(1 - w_{2j}^+ + 2w_{1j}^+(w_{1j}^+ - 1))\right)/P_j - D_{1j}^2 \\ &= -0.07732 \end{aligned}$$

For $j = 2$, $y_j = 0.5$, so that

$$\begin{aligned} D_{1j} &= (Pd_j^-(0.5 - w_{1j}^-) + Pd_j^+(0.5 - w_{1j}^+))/P_j \\ &= 0.04244 \end{aligned}$$

and

$$\begin{aligned} D_{2j} &= \left(Pd_j^-(0.25 - w_{2j}^- + 2w_{1j}^-(w_{1j}^- - 0.5)) + Pd_j^+(0.25 - w_{2j}^+ + 2w_{1j}^+(w_{1j}^+ - 0.5))\right)/P_j - D_{1j}^2 \\ &= -0.07466 \end{aligned}$$

For $j = 3$, $y_j = 0$, so that

$$\begin{aligned} D_{1j} &= (Pl_j^-(0 - w_{1j}^-) + Pl_j^+(0 - w_{1j}^+))/P_j \\ &= -0.33839 \end{aligned}$$

and

$$\begin{aligned} D_{2j} &= (Pl_j^-(0 - w_{2j}^- + 2w_{1j}^-(w_{1j}^- - 0)) + Pl_j^+(0 - w_{2j}^+ + 2w_{1j}^+(w_{1j}^+ - 0)))/P_j - D_{1j}^2 \\ &= -0.07184 \end{aligned}$$

(f) The updated rating and RD on the standardized scale are

$$\begin{aligned} \sigma' &= \sqrt{\frac{1}{1/0.4606^2 - (-0.07732 - 0.07466 - 0.07184)}} = 0.4500 \\ \mu' &= 2.3028 + (0.4500)^2(0.39739 + 0.04244 - 0.33839) = 2.3233 \end{aligned}$$

(to greater number of significant digits, $\sigma' = 0.450006$ and $\mu' = 2.323361$)

(g) The ratings converted to the Elo scale are therefore

$$\begin{aligned} r' &= 173.7(2.323361) + 1500 = 1903.568 \approx 1904 \\ RD' &= 173.7(0.450006) = 78.16604 \approx 78 \end{aligned}$$

At the start of the next rating period, the initial rating to use for the player would be 1903.568, but the initial RD would reflect the extra uncertainty from the passage of time; the initial RD would be

$$RD = \sqrt{78.16604^2 + 25^2} = 82.06662 \approx 82.$$